Int J Theor Phys (2010) 49: 2805–2814
DOI 10.1007/s10773-010-0473-4

Neutrino Oscillations in the Robertson-Walker Metric and the Cosmological Blue Shift of the Oscillation Length

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Received: 13 June 2010 / Accepted: 18 August 2010 / Published online: 2 September 2010

Abstract The Doppler effect of the neutrino oscillation length in the flat space is discussed. When the observer travels away from the source, the oscillation length decreases. Moreover, the mass neutrino oscillation in the Robertson-Walker metric is studied, and the general equations of the oscillation phases are given. We obtain the relation between the phase and the scale factor. The expansion of the universe leads to the reduction of the oscillation length (blue shift), which is contrary to the red shift of the light ray.

Keywords Neutrino oscillation · R-W metric · Expansion of the universe · Oscillation length

1 Introduction

Mass neutrino mixing and oscillations were proposed by Pontecorvo [1]. Mikheyev, Smirnov and Wolfenstein (MSW for short) explored the effect of transformation of one neutrino flavor into another in a medium with varying density [2, 3]. Since the mass neutrino was confirmed by Super-Kamiokande neutrino experiment [4], the consideration of the mass neutrino oscillations has been a hot topic. There have been many theoretical and experimental studies about the neutrino oscillations. Then, the neutrino oscillations in the flat space time were extended to the cases in the curved space-time [5–16]. Furthermore, some alternative mechanisms have been proposed to account for the gravitational effect on the neutrino oscillation [19–21]. The inertial effects on neutrino oscillations were also called attention [22, 23]. As a further theoretical exploration, neutrino oscillations in space-time with both curvature and torsion [24–26] have been studied.

In recent years, the researches about the neutrino oscillation have been made new progress. A further mechanism to generate pulsar kicks, which was based on the spin flavor conversion of neutrinos propagating in a gravitational field, and the neutrino geometrical

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optics in gravitational field (in particular in a Lense-Thirring background), have been proposed by Lambiase [27, 28]. Some publications were centered on the theoretical study and experimental measurement of the mixing angle θ_{13} [29–31]. And CP violation in neutrino oscillations were considered by some authors [32–35]. In addition, Cuesta and Lambiase studied the neutrino mass spectrum [36]. Akhmedov, Maltoni and Smirnov presented the neutrino oscillograms for different oscillation channels and discussed the effects of non-vanishing 1-2 mixing [37].

Mass neutrinos play prominent roles in both particle physics and cosmology, such as the possible origins for neutrino masses and mixings and the implications of this physics for cosmology, as well as the Big Bang Nucleosynthesis (BBN) and bounds on active-sterile neutrino mixing [38]. The model for neutrino masses that results in a new dark matter [39] and dark energy [40], as well as induced inflation [41], has been called much attention recently. Mass neutrino oscillations in large scale universe is also an interested topic in both particle physics and cosmology [42–48] and astrophysics [49], although the possible obtained results are very preliminary at present stage. It would be meaningful to connect the neutrino oscillation with the large scale universe observation, such as the constraints on the neutrino mass and the cosmological constant from large scale structure observations [50]. According to the standard cosmology, the neutrinos are rather abundant in the universe, as abundant as photons and much more abundant than baryons by a factor of about 10^9. Hence the mass density of the neutrinos might be larger than that of the baryons even if the mass of neutrino is as small as 1 keV. Thus the mass neutrinos could induce many interesting astrophysical effects and even make it to be a dominant role in the evolution of the universe [44, 51–53].

In this paper, firstly, we study the Doppler effect of the neutrino oscillation length in flat space-time. The oscillation length decreases as the observer travels away from the source and the blue shift takes place, unlike the case of the red shift of light wave.

Then, the cosmological blue shift of the oscillation length is discussed. With the expansion of the universe, the energy of the neutrino decreases when the neutrino is regarded as a high energy particle, $E \gg m$. Consequently, the oscillation length shortens because the length is in proportion to the energy.

An important probe of the universe at an early time is the oscillation phase itself, Φ_{kj}. At last, we will exploit Robertson-Walker metric to compute the neutrino oscillation phase, which is a bit more complicated in the expanding universe. We calculate the phase along the null geodesic, which is adopted widely in the standard treatment [8, 9, 54–58]. We give the relation between the phase and the scale factor. The oscillation length decreases and the blue shift occurs with the expansion of the universe. For the reason of simplicity, we confine our treatment in two generation neutrinos (electron and muon).

The paper is organized as the follows. In Sect. 2, we briefly review the standard treatment of neutrino oscillation in the flat space-time. In Sect. 3, the Doppler effect of oscillation length is studied in the flat space. In Sect. 4, we discuss cosmological blue shift of the oscillation length. In Sect. 5, we study the neutrino oscillation in R-W metric in detail. At last, the conclusion and discussion are given. Throughout the paper, the units $G = c = \hbar = 1$ are used.

2 Neutrino Oscillation in Flat Space-Time

In a standard treatment, the flavor eigenstate $|\,\nu_\alpha\rangle$ is a superposition of the mass eigenstates $|\,\nu_k\rangle$, i.e. [8, 9]

$$|\nu_\alpha(x,t)\rangle = \sum_k U_{\alpha k} \exp[-i\Phi_k]|\nu_k\rangle, \tag{1}$$

where

$$\Phi_k = E_k t - \vec{p}_k \cdot \vec{x}, \quad (k = 1, 2), \tag{2}$$

and the matrix elements $U_{\alpha k}$ comprise the transformation between the flavor and mass bases. E_k and $\vec{p}_k$ are the energy and momentum of the mass eigenstates $|\nu_k\rangle$, and they are different for different mass eigenstates. If the neutrino produced at a space-time point $A(t_A, \vec{x}_A)$ and detected at $B(t_B, \vec{x}_B)$, the expression for the phase in (2), which is suitable for application in a curved space-time, is [8, 59]

$$\Phi_k = \int_A^B p_\mu^{(k)} dx^\mu, \tag{3}$$

where

$$p_\mu^{(k)} = m_k g_{\mu\nu} \frac{dx^\nu}{ds}, \tag{4}$$

is the canonical conjugate momentum to the coordinate x^μ and m_k is the rest mass corresponding to mass eigenstate $|\nu_k\rangle$. $g_{\mu\nu}$ and ds are metric tensor and the line element, respectively.

The following assumptions are often applied in the standard treatment [54, 55]: (1) The mass eigenstates are taken to be the energy eigenstates, with a common energy; (2) up to $O(m/E)$, there is the approximation $E \gg m$; (3) a massless trajectory is assumed, which means that the neutrino travels along the null trajectory. In the case of two neutrinos mixing $\nu_e - \nu_\mu$, we can write

$$\nu_e = \cos\theta \nu_1 + \sin\theta \nu_2, \, \nu_\mu = -\sin\theta \nu_1 + \cos\theta \nu_2. \tag{5}$$

Here θ is the vacuum mixing angle. The oscillation probability that the neutrino produced as $|\nu_e\rangle$ is detected as $|\nu_\mu\rangle$ is [56]

$$P(\nu_e \to \nu_\mu) = |\langle \nu_e | \nu_\mu(x, t)\rangle|^2 = \sin^2(2\theta) \sin^2\left(\frac{\Phi_{kj}}{2}\right), \tag{6}$$

where, $\Phi_{kj} = \Phi_k - \Phi_j$, is the phase difference. From the standard treatment of the neutrino oscillation [8–10], the standard result for the phase is

$$\Phi_k \simeq \frac{m_k^2}{2E_0} |\vec{x}_B - \vec{x}_A|. \tag{7}$$

Here E_0 is the energy for a massless neutrino. So, the phase difference responsible for the oscillation and the oscillation length are given respectively by

$$\Phi_{kj} \simeq \frac{\Delta m_{kj}^2}{2E_0} |\vec{x}_B - \vec{x}_A|, \tag{8}$$

$$L_{OSC} = \frac{4\pi E_0}{\Delta m_{kj}^2}, \tag{9}$$

where $\Delta m_{kj}^2 = m_k^2 - m_j^2$. From (9), the oscillation length is in proportion to its energy.

3 Doppler Effect in the Flat Space-Time

The Doppler effect of the light-wave is familiar. We will discuss the Doppler effect of the neutrino oscillation length in this section. From (9), the oscillation length is in proportion to its energy E. The 4-velocities of the observer and source of neutrino are U^a and V^a respectively. The neutrino is produced by point p and detected by point q. If the 4-momentum of the neutrino is P^a, the energy of the neutrino measured by V^a and U^a are respectively, $E = -P^a V_a$, $E' = -P^a U_a$. P^a and U_a can be decomposed by V^a at p, respectively

$$P^a = EV^a + p^a, \tag{10}$$

$$U_a = \gamma V_a + \gamma u_a, \tag{11}$$

where $\gamma \equiv -V^a U_a$. So, we can obtain E' as

$$\begin{aligned} E' &= -P^a U_a = -(EV^a + p^a)(\gamma V_a + \gamma u_a) \\ &= \gamma E - \gamma p^a u_a = \gamma E - \gamma u \cos\theta \sqrt{E^2 - m^2}, \end{aligned} \tag{12}$$

where θ is the angle between p^a and u^a. Because the neutrino is a high energy particle, $m \ll E$, we have

$$\sqrt{E^2 - m^2} \simeq E - \frac{m^2}{2E}. \tag{13}$$

Therefore, E' becomes to

$$E' \simeq \gamma E - \gamma u \cos\theta \left(E - \frac{m^2}{2E} \right). \tag{14}$$

If the observer travels away from the source, $\theta = 0$, we get

$$E' \simeq \gamma E - \gamma u \left(E - \frac{m^2}{2E} \right) = E\sqrt{\frac{1-u}{1+u}} + \gamma u \frac{m^2}{2E} < E. \tag{15}$$

This is only right for the high energy particle, $m \ll E$. For the photon $m = 0$, $E = \hbar\omega$, we can get the equation of the Doppler effect of the light-wave. According to (9), the oscillation length is in proportion to its energy and we have, $L'_{OSC} < L_{OSC}$. Thus, the blue shift occurs as the observer travels away from the source, which is unlike the case of the red shift of light wave. The main reason is that the oscillation length is in proportion to its energy. The wavelength of the light is inversely proportional to the energy $\lambda = \frac{hc}{E}$. If the high energy condition is considered, the term in (15), $\gamma u \frac{m^2}{2E}$, can be omitted. The ratio of L'_{OSC} and L_{OSC} is approximated by $\frac{L'_{OSC}}{L_{OSC}} \simeq \sqrt{\frac{1-u}{1+u}}$. The differences between them have to do with the velocity of the observer. If $u = 0.6c$, L'_{OSC} is approximately a half of L_{OSC}.

4 Cosmological Blue Shift of the Oscillation Length

The Robertson-Walker metric can be written as

$$ds^2 = -dt^2 + R^2(t)\left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \right]. \tag{16}$$

The neutrinos produced by Galaxy A at point p_1 and detected by Galaxy B at point p_2 travel along the geodesic. The energy measured by A is

$$E_1 = -mg_{00}\frac{dt}{ds} = m\frac{dt}{ds}\bigg|_{p_1}, \tag{17}$$

where s is the affine parameter. The equations of the geodesic satisfy

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu_{\nu\sigma}\frac{dx^\nu}{ds}\frac{dx^\sigma}{ds} = 0. \tag{18}$$

In a radial direction, $d\theta = 0, d\varphi = 0$, we set $\mu = 0$ and obtain

$$\frac{d^2t}{ds^2} + \frac{R\dot{R}}{1-kr^2}\left(\frac{dr}{ds}\right)^2 = 0, \tag{19}$$

where $\dot{R} = dR/dt$. According to the mass shell condition, we have

$$m^2 = g^{\mu\nu}p_\mu p_\nu = g^{00}p_0p_0 + g^{11}p_rp_r. \tag{20}$$

Substituting $E = p_0 = mdt/ds$ and $p_r = mg_{11}dr/ds$, (20) can be written as

$$-\left(\frac{dt}{ds}\right)^2 + \frac{R^2}{1-kr^2}\left(\frac{dr}{ds}\right)^2 = 1. \tag{21}$$

Substituting (19) and $dt/ds = E/m$, we have

$$\frac{1}{m}\frac{dE}{ds} + \frac{1}{R}\frac{dR}{ds}\frac{m}{E}\left[\left(\frac{E}{m}\right)^2 + 1\right] = 0. \tag{22}$$

It is difficult to exactly integrate this equation. We, however, can discuss as following. Because the neutrinos are relativistic particles, $E \gg m$, 1 in bracket can be negligible. Then, (22) is approximated by

$$\frac{dE}{ds} + \frac{E}{R}\frac{dR}{ds} = 0. \tag{23}$$

This equation can be integrated easily

$$E = \frac{E_0}{R}, \tag{24}$$

where E_0 is an integral constant. Then, according to the relation between the oscillation length (L_{OSC}, abbreviated by L) and energy, $L = \frac{4\pi E}{\Delta m^2}$, we obtain

$$L = \frac{L_0}{R}. \tag{25}$$

The above Equation can be interpreted that with the expansion of the universe the oscillation length of every neutrino is proportionally shortened and the blue shift occurs. Equation (25) is used for point p_1 and p_2

$$\frac{L_2}{L_1} = \frac{R(t_1)}{R(t_2)}. \tag{26}$$

5 Neutrino Oscillation in the Robertson-Walker Metric

In this section, $\eta_{\mu\nu} = diag(+1, -1, -1, -1)$ is used. The Robertson-Walker metric can be written as

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right], \tag{27}$$

where $k = 1, 0, -1$ and t is universe time. The relevant components of the canonical momentum of the k^{th} massive neutrino in (4) are

$$\begin{aligned} p_t^{(k)} &= p_0^{(k)} = m_k g_{00}\frac{dt}{ds}; \\ p_r^{(k)} &= m_k g_{11}\frac{dr}{ds}. \end{aligned} \tag{28}$$

The mass-shell condition is [8]

$$m_k^2 = g_{\mu\nu} p^{(k)\mu} p^{(k)\nu}. \tag{29}$$

The phase along the radial direction from point A to point B is given by [10]

$$\begin{aligned} \Phi_k &= \int_A^B g_{\mu\nu} p^{(k)\mu} dx^\nu \\ &= \int_A^B \left(g_{00} p^{(k)0} + g_{11} p^{(k)r}\frac{dr}{dt}\right) dt. \end{aligned} \tag{30}$$

In the standard treatment of the neutrino oscillation, the neutrino is usually supposed to travel along the null [8, 9, 54–58]. Following the standard treatment, we will calculate the phase along the light-ray trajectory from A to B in this section. The radial null geodesic is

$$0 = g_{00}dt^2 + g_{11}dr^2. \tag{31}$$

The phase can be written as

$$\Phi_k = \int_A^B \left(g_{00} p^{(k)0} + g_{11} p^{(k)r}\frac{dr}{dt}\right) dt = \int_A^B \left(p_0^{(k)} + g_{11} p^{(k)r}\frac{dr}{dt}\right) dt. \tag{32}$$

From the mass-shell condition (29), we have

$$g_{11} p^{(k)r} = -R(t)\frac{\sqrt{p_0^{(k)2} - m_k^2}}{\sqrt{1-kr^2}}. \tag{33}$$

It is assumed that the mass eigenstates are taken to be the energy eigenstates, with a common energy in the standard treatment. The equal energy assumption is considered to be correct by some authors [17, 18, 60] and studied carefully in papers [11, 61, 62]. In addition, it is adopted widely in many literatures, for example [8–10, 63]. In order to keep pace with Symbol in Sect. 2, E is instead of p_0 to represent the common energy of different mass eigenstates in the latter part of this paper. Applying the relativistic condition using the energy E as a reference value, i.e., $m_k \ll E$ [8, 9], the following relation holds

$$g_{11} p^{(k)r} \simeq -\frac{R(t)}{\sqrt{1-kr^2}}\left(E - \frac{m_k^2}{2E}\right). \tag{34}$$

Then, the phase in (32) is approximated by

$$\Phi_k \simeq \int_A^B \frac{m_k^2}{2E} dt. \tag{35}$$

Substituting the relation between E and t (24) in Sect. 4, Φ_k^{null} can be written as

$$\Phi_k = \int_A^B \frac{m_k^2}{2E_0} R(t) dt = \frac{m_k^2}{2E_0} \int_A^B R(t) dt. \tag{36}$$

E_0 represents the energy of the neutrino today. The specific expression of $R(t)$ can be obtained by universe dynamic equation. Then, the phase shift which determines the oscillation is

$$\Phi_{kj} = \frac{\Delta m_{kj}^2}{2E_0} \int_A^B R(t) dt, \tag{37}$$

where $\Delta m_{kj}^2 = m_k^2 - m_j^2$. In order to discuss conveniently, we adopt the differential form of the (37)

$$d\Phi_{kj} = \frac{\Delta m_{kj}^2}{2E_0} R(t) dt. \tag{38}$$

According to radial null geodesic equation, there is

$$\frac{dt}{R(t)} = \frac{dr}{\sqrt{1 - kr^2}}. \tag{39}$$

So, (38) can be rewritten as

$$d\Phi_{kj} = \frac{\Delta m_{kj}^2}{2E_0} \frac{R^2(t)}{\sqrt{1 - kr^2}} dr. \tag{40}$$

The proper distant is

$$dl = \sqrt{-g_{11}} dr = \frac{R(t)}{\sqrt{1 - kr^2}} dr. \tag{41}$$

Here, the proper distance is the distance between two galaxies at the instant t. Thus, (40) becomes to

$$d\Phi_{kj} = \frac{\Delta m_{kj}^2}{2E_0} R(t) dl. \tag{42}$$

We can obtain the proper oscillation length, which is defined by the proper distance as the phase shift Φ_{kj} changing 2π

$$L = \frac{4\pi E_0}{\Delta m_{kj}^2} \frac{1}{R(t)} = \frac{L_0}{R(t)}, \tag{43}$$

where $L_0 = \frac{4\pi E_0}{\Delta m_{kj}^2}$ is the oscillation length today. For the extreme case of radiation dominated universe (energy-momentum tensor, T_{ab}, from the main contribution of the radiation), $k = 0$ and cosmological constant $\Lambda = 0$, $R \propto t^{1/2}$, the length becomes

$$L \propto t^{-1/2}. \tag{44}$$

For the extreme case of material dominated universe (energy-momentum tensor, T_{ab}, from the main contribution of the matter), $k = 0$ and cosmological constant $\Lambda = 0$ (Friedmann Universe), $R \propto t^{2/3}$ [64], the length becomes

$$L \propto t^{-2/3}. \tag{45}$$

For the inflationary universe, $k = 0$, $\Lambda > 0$, $R \propto \exp\sqrt{\frac{\Lambda}{3}}t$ [64], the length is

$$L \propto e^{-\sqrt{\frac{\Lambda}{3}}t}. \tag{46}$$

The relative amount of the blue shift of the oscillation length can be defined as

$$z_{blue} \equiv \frac{L_1 - L_2}{L_1} = 1 - \frac{R(t_1)}{R(t_2)}. \tag{47}$$

The ordinary red shift of the light is

$$z_{red} = \frac{\lambda_2 - \lambda_1}{\lambda_1} = \frac{R(t_2)}{R(t_1)} - 1. \tag{48}$$

So, the blue shift can be expressed as

$$z_{blue} = \frac{z_{red}}{1 + z_{red}}. \tag{49}$$

This result shows that the oscillation length shortens with the expansion of the universe, which is in accordance with (25) obtained in Sect. 4. In the early universe, the scale factor is very small and leads the oscillation length to be very large, which indicates the neutrino to oscillate very weakly. Neutrino oscillations are now becoming more and more intense than in the past, because the recent observation of universe showed that the expansion of our universe is accelerating [65–67].

6 Conclusion and Discussion

Mass neutrinos which may be the main ingredient of the hot dark matter play an important role in the evolution of the universe. In this paper, we discuss the Doppler effect, cosmological blue shift of the neutrino oscillation length and the oscillation phase in R-W metric. When the observer travels away from the source, the blue shift of the oscillation length will take place. The oscillation length is in direct proportion to its energy. The neutrinos as high-energy relativistic particles, their energies decrease with the expansion of the universe, which cause the reduction of the oscillation length. Unlike in the Minkowski space, the calculation of the oscillation phase is a bit more complicated in the expanding universe. Considering the time dilation of our clocks compared to the early universe clocks, we get the very simple results for oscillation phase,

$$\Phi_k = \frac{m_k^2}{2E_0} \int_A^B R(t)dt, \tag{50}$$

where the integrand is the scale-factor $R(t)$. This indication shows that neutrino oscillation phase is only determined by $R(t)$, nothing to do with other metric quantities in Robertson-Walker metric. It is noted that in (50) the phase is the integral of the variable with respect to time t, not to r. In R-W metric the space is homogeneous and isotropic at any given instant of time, but $\partial/\partial t$ is not killing. However, in the Schwarzschild space-time, $\partial/\partial t$ is killing, and the space is not homogeneous along the r direction. So, the phase is the integral of the variable with respect to r, not to t in the Schwarzschild space-time [68].

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